M11: Culvert Hydraulics

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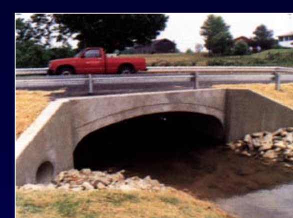
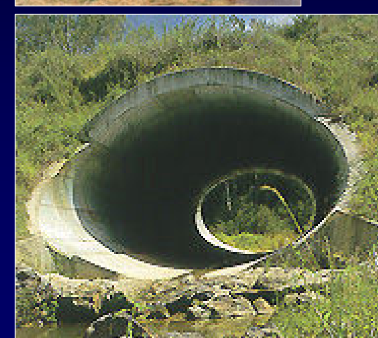
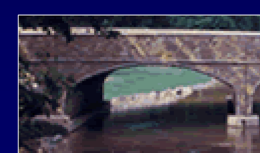
Culvert Systems

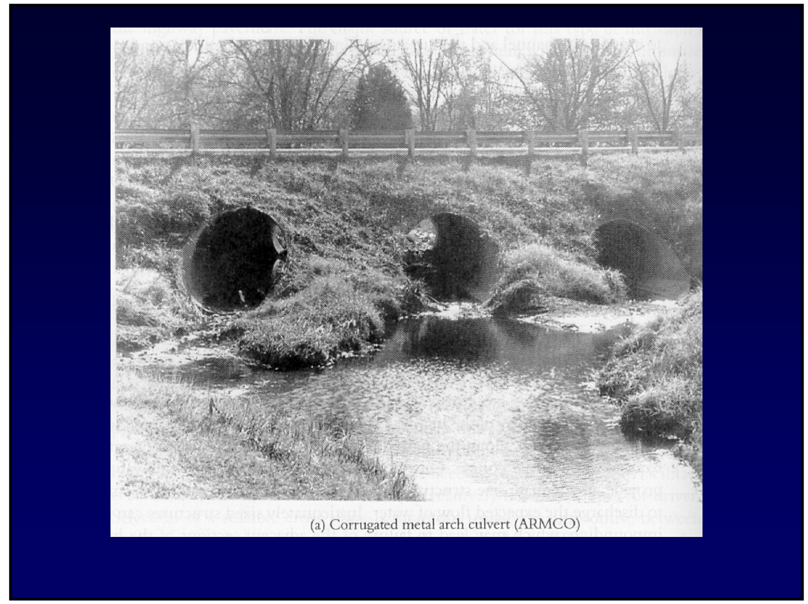
- Culverts typically used in roadway crossings and detention pond outlets.
- Headwater elevation – water surface elevation just upstream of the culvert
- Tailwater elevation – water surface elevation just downstream of the culvert
- Analysis typically for:
  - Size, shape and number of new or additional culverts needed to pass a design discharge
  - Hydraulic capacity of existing culvert system
  - Upstream flood level at an existing culvert system resulting from a specific discharge rate
  - Hydraulic performance curves for a culvert system (which are used to assess hydraulic risk at a crossing or as input for another hydraulic or hydrologic model

Culvert Flow

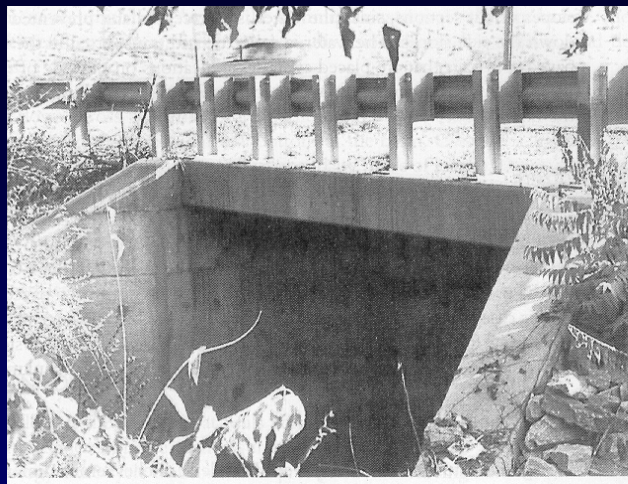








(a) Corrugated metal arch culvert (ARMCO)



(b) Rectangular (box) culvert

Figure I-1--Typical Concrete Pipe Culvert

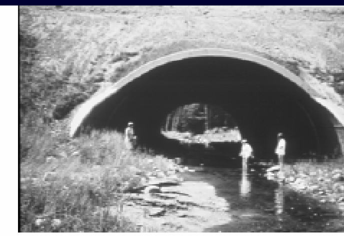


Figure I-2--Long Span Culvert

From: FHWA. *Hydraulic Design of Highway Culverts*.

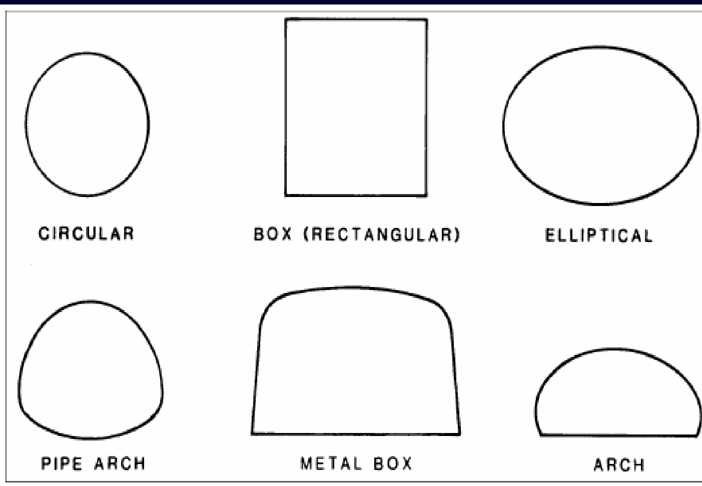


Figure I-4--Commonly Used Culvert Shapes

From: FHWA. *Hydraulic Design of Highway Culverts*.

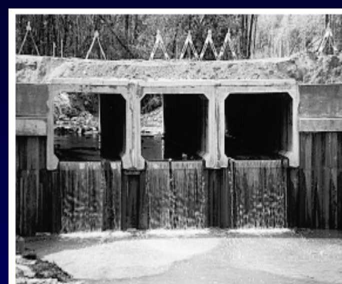


Figure I-5--Precast Concrete Box Culvert (American Concrete Pipe Association)

Figure I-6 – Corrugated Metal Arch (Contech)

From: FHWA. *Hydraulic Design of Highway Culverts*.

Culvert Hydraulics: Control Type

- Culverts act as a significant constriction to flow and are subject to a range of flow types, including both gradually varied and rapidly varied flow.
- Simplify by control type:
  - Outlet Control Assumption:
    - Computes the upstream headwater depth using conventional hydraulic methodologies that consider the predominant losses due to culvert barrel friction
    - Also includes minor entrance and exit losses.
    - Tailwater condition has important effect on culvert system.
  - Inlet Control Assumption:
    - Computes upstream headwater depth resulting from constriction at the culvert entrance
    - Neglects culvert barrel friction, tailwater elevation and other minor losses.
- The controlling headwater depth is the large of the computed inlet and outlet control headwater depths (since a single culvert may at times operate under each of the two control types.

Culvert Hydraulics: Outlet Control

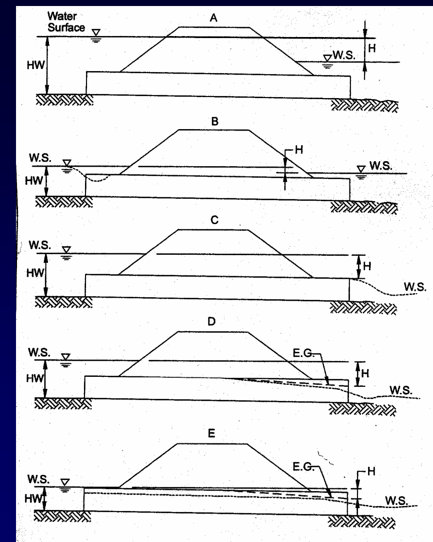


Figure 4-1: Outlet Control Flow Conditions

Culvert Hydraulics: Outlet Control

- Headwater depth is found by summing the tailwater depth, entrance minor loss, exit minor loss and friction losses along the culvert barrel.
- Energy basis for solving the outlet control headwater (HW) for a culvert under inlet control is given by the basic energy equation, rewritten for culvert terms.

$$HW_0 + \frac{V_u^2}{2g} = TW + \frac{V_d^2}{2g} + H_L$$

Where HW_0 = headwater depth above outlet invert (length)
V_u = approach velocity (length/time)
TW = tailwater depth above outlet invert (length)
V_d = exit velocity (length/time)
H_L = sum of all losses (entrance minor loss [H_E] + barrel friction losses (H_F) + exit loss [H_O] + other losses), (length)

Culvert Hydraulics: Outlet Control

- When the culverts connect ponds or other waterbodies with negligible velocity on the upstream and downstream, the equation simplified to:

$$HW_0 = TW + H_L$$

- Culverts are often hydraulically short (meaning that uniform depth will not be achieved during water's passage through the culvert).
  - Solved using the gradually-varied flow analysis techniques.

Culvert Hydraulics: Outlet Control

- Entrance losses due to contraction of flow as it enters the culvert.
- Entrance losses are a function of barrel velocity head just inside the entrance, with the smoother entrances having the lowest entrance loss coefficients.
- Entrance losses expressed using the following equation:

$$H_E = k_e\left(\frac{V^2}{2g}\right)$$

Where H_E = entrance loss (length)
k_e = entrance loss coefficient
V = velocity just inside barrel entrance (length/time)
g = gravitational constant (length/time^2)

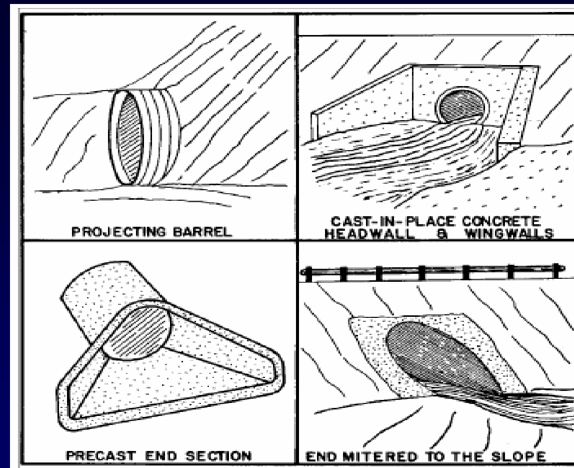


Figure I-7 –Four Standard Inlet Types (schematic)

From: FHWA. *Hydraulic Design of Highway Culverts*.

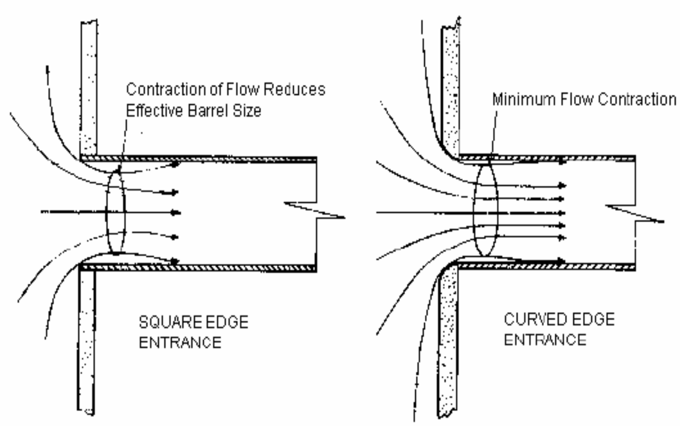


Figure I-8 - - Entrance Contraction (schematic).

From: FHWA. *Hydraulic Design of Highway Culverts*.

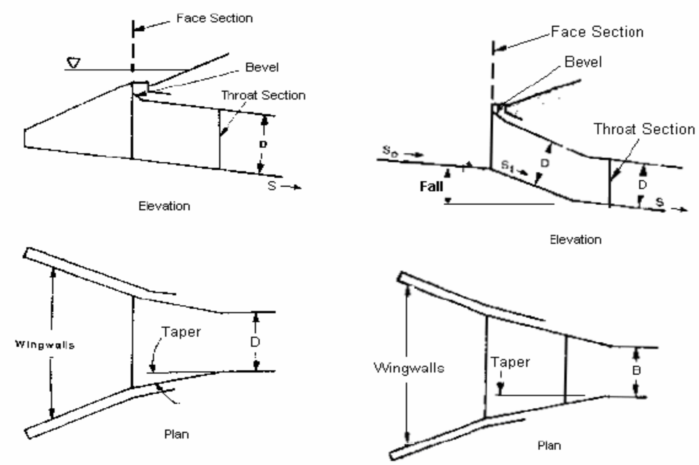


Figure I-9 - - Side-tapered inlet **Figure I-10 - - Slope-tapered inlet**

From: FHWA. *Hydraulic Design of Highway Culverts*.

Culvert Hydraulics: Outlet Control

Culvert Type	Entrance Type and Description	Entrance Loss Coefficient, k_e
Pipe, Concrete	Projecting from fill, socket end (groove-end)	0.3
	Projecting from fill, square cut end	0.5
	Headwall or headwall with wingwalls	
	Socket end of pipe (groove-end)	0.2
	Square edge	0.5
	Rounded (radius = 1/12 D)	0.2
	Mitered to conform to fill slope	0.7
	End-section confirming to fill slope	0.5
	Beveled edges, 33.7° or 45° levels	0.2
	Side or slope-tapered inlet	0.2

Culvert Hydraulics: Outlet Control

Culvert Type	Entrance Type and Description	Entrance Loss Coefficient, k_e
Pipe or Pipe Arch, Corrugated Metal	Projecting from fill (no headwall)	0.9
	Headwall or headwall and wingwalls square-edge	0.5
	Mitered to conform to fill slope, paved or unpaved edge	0.7
	End-section confirming to fill slope	0.5
	Beveled edges, 33.7° or 45° levels	0.2
	Side or slope-tapered inlet	0.2

Culvert Hydraulics: Outlet Control

Culvert Type	Entrance Type and Description	Entrance Loss Coefficient, k_e
Box Culvert	Headwall parallel to embankment (no wingwalls)	
	Square-edged on 3 edges	0.5
	Rounded on 3 edges (to radius of 1/12 barrel dimension or beveled edges on 3 sides)	0.2
	Wingwalls at 30° to 75° barrel	
	Square-edged at crown	0.5
	Crown edge rounded (to radius of 1/12 barrel dimension, or beveled top edge)	0.2
	Wingwall at 10° to 25° to barrel	
	Square-edged at crown	0.5
	Wingwalls parallel (extension of sides)	
	Square-edged at crown	0.5
	Side or slope-tapered inlet	0.2

Culvert Hydraulics: Outlet Control

- Exit loss is an expansion loss.
- Function of change in velocity head that occurs at the discharge end of the culvert.
- Exit losses expressed using the following equation:

$$H_O = 1.0\left(\frac{V^2}{2g} - \frac{V_d^{\,2}}{2g}\right)$$

Where H_O = exit loss (length)
V_d = velocity of outfall channel
V = velocity just inside end of culvert barrel (length/time)
g = gravitational constant (length/time^2)

- When discharge is negligible, exit loss equal to barrel velocity head.
- Typically solved using gradually-varied flow analysis.

Culvert Hydraulics: Inlet Control

- When operating under inlet control, hydraulic control section is culvert entrance.
- Typically, the friction and minor losses in the culvert are not as significant.
- Critical depth normally occurs at or near the inlet, and flow downstream of the inlet are supercritical.
- Three types of inlet control:
  - Unsubmerged – For low discharge conditions, the culvert entrance acts as a weir.
  - Submerged – When the culvert is fully submerged, the inlet operates as an orifice.
  - Transitional – Region just above the unsubmerged zone and below the fully submerged zone.

Culvert Hydraulics: Inlet Control

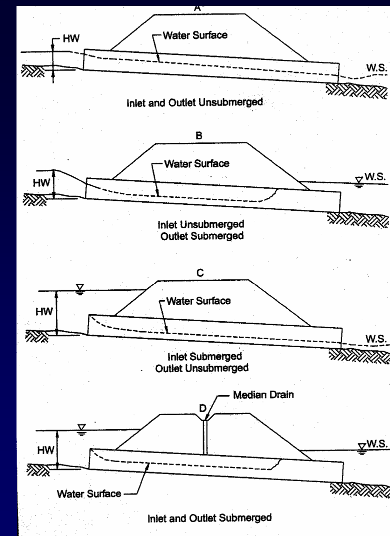


Figure 4-2: Inlet Control Flow Conditions

Culvert Hydraulics: Inlet Control

- Unsubmerged Flow
  - Two equations possible (typical to use the 2nd one for hand calcs).
  - Form 1: $\frac{HW_i}{D} = \frac{H_c}{D} + K\left[\frac{Q}{AD^{0.5}}\right]^M - 0.5S$
  - Form 2: $\frac{HW_i}{D} = K\left[\frac{Q}{AD^{0.5}}\right]^M$

Where
HW_i = headwater depth above the control section invert (length)
D = interior height of culvert barrel (length)
H_c = specific head at critical depth, $y_c + V_c^2/2g$ (length/time)
Q = culvert discharge ($length^3/time$)
A = full cross-sectional area of the culvert barrel ($length^2$)
S = culvert barrel slope
K,M = constants from table

Mitered inlets: use slope correction factor of +0.7S instead of -0.5S

Culvert Hydraulics: Inlet Control

- Submerged Flow
  - Equation for submerged (orifice) flow:

$$\frac{HW_i}{D} = c\left[\frac{Q}{AD^{0.5}}\right]^2 + Y - 0.5S$$

Mitered inlets: use slope correction factor of +0.7S instead of -0.5S

Where
HW_i = headwater depth above the control section invert (length)
D = interior height of culvert barrel (length)
H_c = specific head at critical depth, $y_c + V_c^2/2g$ (length/time)
Q = culvert discharge ($length^3/time$)
A = full cross-sectional area of the culvert barrel ($length^2$)
S = culvert barrel slope
K,M = constants from table
c, Y = constants from table

- Equation for submerged flow applicable when $Q/AD^{0.5} = 4.0$

Coefficients for Inlet Control Design Equations

Shape and Material	Inlet Edge Description	Equation Form	Unsubmerged		Submerged	
			K	M	c	Y
Circular Concrete	Square edge with headwall	1	0.0098	2.0	0.0398	0.67
	Groove end with headwall		0.0078	2.0	0.0292	0.74
	Groove end projecting		0.0045	2.0	0.317	0.69
Circular CMP	Headwall	1	0.0078	2.0	0.379	0.69
	Mitered to slope		0.0210	1.33	0.0463	0.75
	Projecting		0.0340	1.50	0.0553	0.54
Circular	Beveled ring, 45º bevels	1	0.0018	2.50	0.300	0.74
	Beveled ring, 33.7º bevels		0.0018	2.50	0.0243	0.83

Coefficients for Inlet Control Design Equations

Shape and Material	Inlet Edge Description	Equation Form	Unsubmerged		Submerged	
			K	M	c	Y
Rectangular Box	30º to 75º wingwall flares	1	0.026	1.0	0.0385	0.81
	90º and 15º wingwall flares		0.061	0.75	0.0400	0.80
	0º wingwall flares		0.061	0.75	0.0423	0.82
Rectangular Box	45º wingwall flares d = 0.0430	2	0.510	0.667	0.0309	0.80
	18º to 33.7o wingwall flare d = 0.0830		0.486	0.667	0.0249	0.83
Rectangular Box	90º headwall with ¾" chamfers	2	0.515	0.667	0.0375	0.79
	90º headwall with 45º bevels		0.495	0.667	0.0314	0.82
	90º headwall with 33.7º bevels		0.486	0.667	0.0252	0.865

Coefficients for Inlet Control Design Equations

Shape and Material	Inlet Edge Description	Equation Form	Unsubmerged		Submerged	
			K	M	c	Y
Rectangular Box	¾" chamfers, 45º skewed headwall	2	0.522	0.667	0.0402	0.73
	¾" chamfers, 30º skewed headwall		0.533	0.667	0.0425	0.705
	¾" chamfers, 15º skewed headwall		0.545	0.667	0.04505	0.68
	45º bevels; 10º-45º skewed headwall		0.498	0.667	0.0327	0.75
Rectangular Box, ¾" Chamfers	45º non-offset wingwall flares	2	0.497	0.667	0.0339	0.803
	18.4º non-offset wingwall flares		0.493	0.667	0.0361	0.806
	18.4º non-offset wingwall flares, 30º skewed barrel		0.495	0.667	0.0386	0.71

Coefficients for Inlet Control Design Equations

Shape and Material	Inlet Edge Description	Equation Form	Unsubmerged		Submerged	
			K	M	c	Y
Rectangular Box, Top Bevels	45º wingwall flares – offset	2	0.497	0.667	0.0302	0.835
	33.7º wingwall flares – offset		0.493	0.667	0.0252	0.881
	18.4º wingwall flares – offset		0.495	0.667	0.0227	0.887
CM Boxes	90º headwall	1	0.0083	2.0	0.0379	0.69
	Thick wall projecting		0.0145	1.75	0.0419	0.64
	Thin wall projecting		0.0340	1.5	0.0496	0.57
Horizontal Ellipse Concrete	Square edge with headwall	1	0.0100	2.0	0.0398	0.67
	Groove end with headwall		0.0018	2.5	0.0292	0.74
	Groove end projecting		0.0045	2.0	0.0317	0.69

Coefficients for Inlet Control Design Equations

Shape and Material	Inlet Edge Description	Equation Form	Unsubmerged		Submerged	
			K	M	c	Y
Vertical Ellipse Concrete	Square edge with headwall	1	0.0100	2.0	0.0398	0.67
	Groove end with headwall		0.0018	2.5	0.0292	0.74
	Groove end projecting		0.0045	2.0	0.0317	0.69
Pipe Arch 18" Corner Radius CM	90° headwall	1	0.0083	2.0	0.0379	0.69
	Mitered to slope		0.0300	1.0	0.0463	0.74
	Projecting		0.0340	1.5	0.0496	0.57
Pipe Arch 18" Corner Radius CM	Projecting	1	0.0296	1.5	0.0487	0.55
	No bevels		0.0087	2.0	0.0361	0.66
	33.7° bevels		0.0030	2.0	0.0264	0.75

Coefficients for Inlet Control Design Equations

Shape and Material	Inlet Edge Description	Equation Form	Unsubmerged		Submerged	
			K	M	c	Y
Pipe Arch 31" Corner Radius CM	Projecting	1	0.0296	1.5	0.0487	0.55
	No bevels		0.0087	2.0	0.0361	0.66
	33.7° bevels		0.0030	2.0	0.0264	0.75
Arch CM	90o headwall	1	0.0083	2.0	0.0379	0.69
	Mitered to slope		0.0300	1.0	0.0463	0.75
	Thin wall projecting		0.0340	1.5	0.0496	0.57
Circular	Smooth tapered inlet throat	2	0.534	0.555	0.0196	0.89
	Rough tapered inlet throat		0.519	0.64	0.0289	0.90

Coefficients for Inlet Control Design Equations

Shape and Material	Inlet Edge Description	Equation Form	Unsubmerged		Submerged	
			K	M	c	Y
Elliptical Inlet Face	Tapered inlet - beveled edges	2	0.536	0.622	0.0368	0.83
	Tapered inlet - square edges		0.5035	0.719	0.0478	0.80
	Tapered inlet – thin edge projecting		0.547	0.80	0.0598	0.75
Rectangular	Tapered inlet throat	2	0.475	0.667	0.0179	0.97
Rectangular Concrete	Side tapered – less favorable design	2	0.56	0.667	0.0466	0.85
	Side tapered – more favorable design		0.56	0.667	0.0378	0.87
Rectangular Concrete	Slope tapered – less favorable design	2	0.50	0.667	0.0466	0.65
	Slope tapered – more favorable design		0.50	0.667	0.0378	0.71

Hydraulic Operation of Culverts: Simplified

- Hydraulics of culverts can be classified into four categories:
  1. Submerged inlet and outlet
  2. Submerged inlet with full flow but free discharge at the outlet
  3. Submerged inlet with partially full pipe flow
  4. Unsubmerged inlet

Hydraulic Operation of Culverts: Simplified

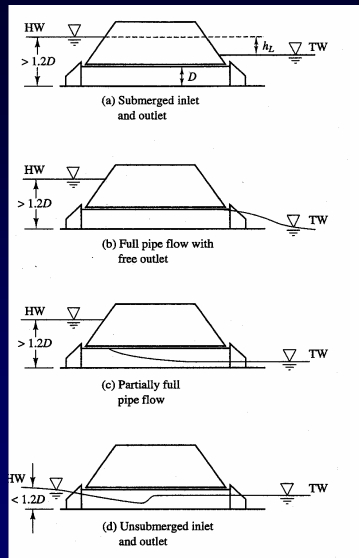


Figure 8.17 Hydraulic operation of culverts.

Culvert Operation: Submerged Inlet and Outlet

- Culvert discharge is primarily affected by tailwater elevation (TW) and the head loss of the culvert (regardless of culvert slope). Culvert flow can be treated as pressure pipe flow. Headloss is sum of culvert head loss and exit and entrance losses.
- Equation for headloss in this culvert:

$$h_L = k_e\left(\frac{V^2}{2g}\right) + \frac{n^2 V^2 L}{R_h^{4/3}} + \frac{V^2}{2g}$$

- Entrance coefficient, k_e, approximately 0.5 for a square-edged entrance and 0.1 for a well-rounded entrance.
- Manning's n: n = 0.013 for concrete; n = 0.024 for corrugated metal pipe.

Culvert Operation: Submerged Inlet and Outlet

- Equation for headloss in this culvert type in a circular culvert:

$$h_L = \left[k_e + \frac{2gn^2 L}{R_h^{4/3}} + 1\right]\left(\frac{8Q^2}{\pi^2 g D^4}\right)$$

Where Q = discharge

D = diameter

R_h = hydraulic radius of the culvert barrel (= D/4 for full-flowing barrel)

Culvert Operation: Submerged Inlet with Free Outlet Discharge

- If the discharge carried in a culvert has a normal depth larger than the barrel height, the culvert will flow full even if the tail water level drops below that of the outlet.
- Discharge is controlled by headloss and level of headwater.
- Equations are same as for the submerged inlet and outlet.

Culvert Operation: Submerged Inlet with Partially Full Pipe Flow

- If the normal depth is less than the barrel height, with the inlet submerged and free discharge at the outlet, a partially full pipe flow condition will normally result.
- The culvert discharge is controlled by the entrance conditions (head water, barrel area, and edge conditions), and the flow is under entrance control.
- Discharge calculated by the orifice equation:

$$Q = C_d A\sqrt{2gh}$$

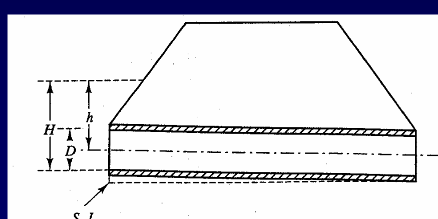
Where
h = hydrostatic head above the center of the pipe opening
A = cross-sectional area
C_d = coefficient of discharge (C_d = 0.62 for square-edged entrance and C_d = 1.0 for well-rounded entrance)

Culvert Operation: Unsubmerged Inlet

- When the hydrostatic head at the entrance is less than 1.2 D, air will break into the barrel.
- No longer pressure pipe flow.
- Culvert slope and barrel wall friction will dictate flow.
- Due to a sudden reduction of water area at entrance, flow usually enters the culvert in supercritical condition.
- Critical depth takes place at the entrance of the barrel.
- If friction is sufficient, depth of flowing water increases.
- Depending on tailwater elevation, supercritical flow may convert to subcritical flow through hydraulic jump.
- Water surface profile calculated using gradually-varied flow equations.

Culvert Hydraulics: Example

- A corrugated steel pipe is used as a culvert that must carry a flow rate of 5.3 m^3/sec and discharge into the air. At the entrance, the maximum available head water is 3.2 m above the culvert invert. The culvert is 35 m long and has a square-edged entrance and slope of 0.003. Determine the diameter of the pipe.



Culvert Hydraulics: Example

- Of the four types of culvert hydraulics, determine the type.
  - Not unsubmerged inlet.
  - Not submerged outlet.
  - Check for submerged inlet with partially full flow and pressurized pipe flow.
- Assume full pipe flow:

$$h_L = H - D + S_o L$$
$$h_L = 3.2 - D + (0.003)(35m) = 3.305 - D$$
$$h_L = \left[k_e + \frac{2gn^2 L}{R_h^{4/3}} + 1\right]\left(\frac{8Q^2}{\pi^2 g D^4}\right)$$
$$h_L = \left[0.5 + \frac{2g(0.024)^2(35m)}{(D/4)^{4/3}} + 1\right]\left(\frac{8(5.3m^3/\sec)^2}{\pi^2 g D^4}\right)$$

Culvert Hydraulics: Example

- Assume full pipe flow:

$$3.305 - D = \left[0.5 + \frac{2g(0.024)^2(35m)}{(D/4)^{4/3}} + 1\right]\left(\frac{8(5.3m^3/\sec)^2}{\pi^2 g D^4}\right)$$

$$3.305 = D + \left(1.5 + \frac{2.51}{D^{4/3}}\right)\left(\frac{2.321}{D^4}\right)$$

$$D = 1.395m \approx 1.4m$$

Culvert Hydraulics: Example

- Assume partially full pipe flow:
  - Discharge controlled by entrance condition only.
  - Head is measured above centerline of pipe.

$$h + \frac{D}{2} = 3.2m$$

$$h = 3.2 - \frac{D}{2}$$

Culvert Hydraulics: Example

- Assume partially full pipe flow:
  - Discharge controlled by entrance condition only.
  - Orifice Formula (and substituting for h):

$$Q = C_d A\sqrt{2gh}$$

$$5.3m^3/\sec = (0.62)\left(\frac{\pi}{4}D^2\right)\sqrt{2g\left(3.2 - \frac{D}{2}\right)}$$

$$D = 1.24m$$

Resistance to flow in the pipe limits the flow. Therefore, use the diameter calculated with this assumption (submerged inlet and full pipe flow).
D = 1.4 m

Summary of Culvert Flow Conditions:

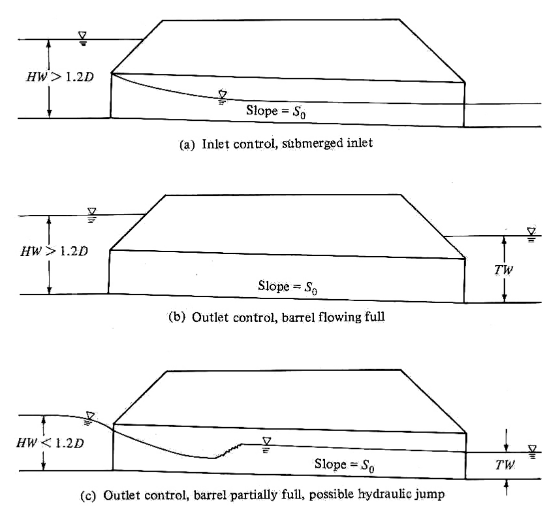


Figure 11–11 Culvert hydraulics.

Prasuhn 1987